FYS-4096 Computational Physics, exercise 8

Return your solution to project `exercise8` under your GitLab group for this course by Friday 5 AM.

Tag the final version with `final` keyword, and make sure to include a file `problems_solved` in the repository. The `problems_solved`-file should be a comma separated list of problems you have solved.

Return your solutions as a single PDF file with maximum file size 5 MB. If your handwriting is inlegible, please type in your solutions with LaTeX.

Problems

Derive the weak form of the following equations

1. ‘$-\frac{1}{2}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$‘, where ‘$\psi(x)$‘ is a real valued function in the interval ‘$x \in [-L, L]$‘ with Dirichlet boundary conditions ‘$\psi(\pm L) = 0$‘. (1 XP)

2. ‘$\nabla^2 u(\mathbf{r}) = f(\mathbf{r})$‘ where the unknown is ‘$u : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$‘. The domain and the boundary conditions are shown in the figure below. (1 XP)

3. ‘$\nabla^2 u(\mathbf{r}) = f(\mathbf{r}) + v(\mathbf{r})$‘ and ‘$\nabla^2 v(\mathbf{r}) = g(\mathbf{r}) + u(\mathbf{r})$‘ where we have two unknowns, ‘$u : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$‘ and ‘$v : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$‘. The domain and the boundary conditions are shown in the figure below. (1 XP)

4. Derive the weak form for the spatial part of following diffusion equation (1 XP)

‘$\partial_t u(t, \mathbf{r}) = \nabla^2 u(t, \mathbf{r}) + f(\mathbf{r})$‘, where ‘$u(t, \mathbf{r}) : \mathbb{R}_+ \times [0, 1]^2$‘ is our unknown scalar field with 2 spatial coordinates and one temporal coordinate. The flux should be zero at the boundary.

5. Radioactive casing (2 XP)

We place a 4 gram sample of pure Cesium-137 inside a hollow steel sphere with inner and outer radii ‘R_{min}‘ and ‘R_{max}‘. The outer surface of the steel sphere is kept at constant temperature of 20°C. Write down a partial differential equation and boundary conditions for the temperature of the steel sphere as a function of distance from its center after a suitable transient time has passed. You may neglect any changes to the sample activity in time. Derive the weak form of the obtained PDE.

Extra: Solve the problem analytically (+ 2 XP)

6. Resident Evil (4 XP)

A minor mistake by a biochem grad student. All he forgot was to close the door of the refrigerator where they kept their samples for various viral strains. An earthquake shook the lab, and due to a series of unfortunate accidents the virus samples got into contact with radioactive samples in the physics lab. A few students and a professor got infected and so the zombie outbreak started.

Your task is to develop a model for the dynamics of human '$H(\mathbf{r}, t)$' and zombie '$Z(\mathbf{r}, t)$' population densities. You can model a closed system, e.g., Australia.

Come up with a few basic properties on how the human and zombie populations/densities interact, e.g., * how the zombie population moves * how does the zombie population increase/decrease with respect to the human population and vice versa (infection rate, kill rate, ...) * how do humans react to zombie population? Do they run away?

Finally, write down (and justify) equations and boundary conditions that govern the time-evolution of human and zombie population densities in your model. Derive the weak form for the spatial part of your equations.

For inspiration, check out *Mathematical Modelling of Zombies* by R. Smith et al.